

CP phase from twisted Higgs VEV in extra dimension

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Abstract

We propose a new mechanism to generate a CP phase originating from a non-trivial Higgs vacuum expectation value in an extra dimension. A twisted boundary condition for the Higgs doublet can produce an extra dimensional coordinate-dependent vacuum expectation value containing a CP phase degree of freedom. With this mechanism, we construct a phenomenological model on S^1 which can naturally explain the origins of the fermion generations, the quark mass hierarchy and the structure of the Cabibbo–Kobayashi–Maskawa matrix with the CP phase. It is noted that the model contains only a single generation for each five-dimensional quark and hence the five-dimensional Yukawa couplings cannot be the origin of the CP phase. The three quark generations can arise in four-dimensions by introducing point interactions on the extra dimension. Our situation is phenomenologically-desirable since all the flavor structures are realized with good precision and almost all dimensionless parameters take values of natural $\mathcal{O}(10)$ magnitudes.

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1 Introduction

Pursuing the origins of the quark mass hierarchy, the flavor mixings, and the three generations of the fermions is one of the important themes in particle physics. Lots of experiments have succeeded in measuring values of the quark masses and the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix with good precision. A complex phase in the CKM matrix has been proposed to explain an origin of the CP violation [1]. The existence of the CP phase has been well established by B physics experiments. The standard model (SM) possesses these structures, though their origin is unknown and some tuning in the Yukawa couplings is required.

In the context of higher dimensional theories, we can utilize a lot of tools whose counterparts are not found in the theory in the four dimensions. Many studies have been done until today based on many ideas [2–18]. In this letter, we propose a new mechanism to produce a CP phase and construct a phenomenological model on S^1 which can naturally explain all the flavor structure of the SM. We mention that in our model, all those properties are derived via geometry of our system. A twisted boundary condition (BC) for the Higgs doublet is found to produce an extra dimensional coordinate-dependent vacuum expectation value (VEV) containing a CP phase degree of freedom. Properties of such type of scalar VEVs have been studied in Refs. [19–28]. Point interactions on S^1 , which are additional boundary points¹ (on S^1), have been studied in Refs [33–37] and we use them to realize the three fermion generations from a single five-dimensional (5D) fermion. It should be emphasized that our model contains only a single generation for each 5D quark and hence the 5D Yukawa couplings have no genuine CP phase. Our mechanism can, however, work to produce a non-trivial CP phase in the CKM matrix via a coordinate-dependent complex phase of the Higgs VEV.² The fermion mass hierarchy is shown to be solved with a scalar VEV which depends on the extra coordinate exponentially.

This letter is organized as follows: In Section 2, we discuss and verify a possibility of the Higgs doublet with a twisted BC to explain the origin of the CP phase in the CKM matrix. In Section 3, we construct a concrete model with point interactions and a scalar singlets whose VEV depends on the extra coordinate exponentially. In Section 4, we search for a set of model parameters where all the flavor structures are realized with good precision and almost all dimensionless parameters take values of $\mathcal{O}(10)$ magnitudes. In Section 5, we summarize our results and discuss some aspects of our model.

2 Position-dependent VEV with twisted boundary condition

In this section, we discuss the property of the VEV of a $SU(2)_W$ doublet scalar H with a twisted boundary condition on S^1 . The action we consider is

$$S_H = \int d^4x \int_0^L dy \left\{ H^\dagger (\partial_M \partial^M + M^2) H - \frac{\lambda}{2} (H^\dagger H)^2 \right\}, \quad (2.1)$$

¹ We can consider a possibility that some terms are localized in boundary points at tree level [29–32].

² In the gauge-Higgs unification model, a similar problem arises because of lack of degree of freedom in the Yukawa sector of an original 5D action. The ways to overcome this point have been studied [38–40].

where M and λ are the bulk mass and quartic coupling, respectively. Since S^1 is a multiply-connected space, we can impose the twisted boundary condition on H as [19–23]

$$H(y + L) = e^{i\theta} H(y). \quad (2.2)$$

Here, we take the range of θ as $-\pi < \theta \leq \pi$. We use a coordinate y to indicate the position in the extra space, L shows the circumference of S^1 , and we choose the metric convention as $\eta_{MN} = \eta^{MN} = \text{diag}(-1, 1, 1, 1, 1)$. The latin indices run from 0 to 3, 5 (or y) and greek ones run from 0 to 3, respectively.

We note that the VEV of $\langle H(y) \rangle$ should be determined by minimizing the functional

$$\mathcal{E}[H] = \int_0^L dy \left\{ |\partial_y H|^2 - M^2 |H|^2 + \frac{\lambda}{2} |H|^4 \right\}, \quad (2.3)$$

because the VEV can possess the y -dependence to minimize the energy. Here, we assume that the four-dimensional (4D) Lorentz invariance is unbroken.

After introducing $\mathcal{H}(y)$ by

$$H(y) = e^{i\frac{\theta}{L}y} \mathcal{H}(y), \quad \mathcal{H}(y + L) = \mathcal{H}(y), \quad (2.4)$$

the functional \mathcal{E} can be rewritten as

$$\mathcal{E}[H] = \mathcal{E}_1[\mathcal{H}] + \mathcal{E}_2[\mathcal{H}], \quad (2.5)$$

$$\mathcal{E}_1[\mathcal{H}] = \int_0^L dy \left\{ |\partial_y \mathcal{H}|^2 + i \frac{\theta}{L} \left((\partial_y \mathcal{H})^\dagger \mathcal{H} - \mathcal{H}^\dagger \partial_y \mathcal{H} \right) \right\}, \quad (2.6)$$

$$\mathcal{E}_2[\mathcal{H}] = \int_0^L dy \left\{ \frac{\lambda}{2} \left(|\mathcal{H}|^2 - \frac{1}{\lambda} \left[M^2 - \left(\frac{\theta}{L} \right)^2 \right] \right)^2 - \frac{1}{2\lambda} \left[M^2 - \left(\frac{\theta}{L} \right)^2 \right]^2 \right\}, \quad (2.7)$$

where \mathcal{E}_1 corresponds to the contribution from the y -kinetic term of \mathcal{H} .

Since $\mathcal{H}(y)$ satisfies the periodic boundary condition, $\mathcal{H}(y)$ can be decomposed as

$$\mathcal{H}(y) = \sum_{n=-\infty}^{\infty} \frac{\vec{a}_n}{\sqrt{L}} e^{i2\pi n \frac{y}{L}}, \quad (2.8)$$

where \vec{a}_n is a two-component $SU(2)_W$ constant vector. Substituting Eq. (2.8) into \mathcal{E}_1 , we obtain the expression

$$\mathcal{E}_1 = \sum_{n=-\infty}^{\infty} \left[\left(\frac{2\pi n + \theta}{L} \right)^2 - \left(\frac{\theta}{L} \right)^2 \right] |\vec{a}_n|^2 \geq 0 \quad (2.9)$$

and we can conclude that the minimum of \mathcal{E}_1 is given by $\mathcal{E}_1 = 0$ when the values of θ and \vec{a}_n satisfy one of the following conditions:

$$(i) \quad -\pi < \theta < \pi \text{ and } \vec{a}_n = 0 \ (n \neq 0) : \quad \mathcal{H} = \frac{\vec{a}_0}{\sqrt{L}}, \quad (2.10)$$

$$(ii) \quad \theta = \pi \text{ and } \vec{a}_n = 0 \ (n \neq 0, -1) : \quad \mathcal{H} = \frac{\vec{a}_0}{\sqrt{L}} \text{ or } \mathcal{H} = \frac{\vec{a}_{-1}}{\sqrt{L}} e^{-i2\pi \frac{y}{L}}, \quad (2.11)$$

where \vec{a}_0 in Eq. (2.10) and \vec{a}_0 or \vec{a}_{-1} in Eq. (2.11) are still undetermined. The functional \mathcal{E}_2 takes the minimum value if the following condition is fulfilled:

$$|\mathcal{H}|^2 = \begin{cases} \frac{1}{\lambda} \left(M^2 - \left(\frac{\theta}{L} \right)^2 \right) & \text{for } M^2 - \left(\frac{\theta}{L} \right)^2 > 0 \\ 0 & \text{for } M^2 - \left(\frac{\theta}{L} \right)^2 \leq 0 \end{cases}. \quad (2.12)$$

Combining the above two results and using the $SU(2)_W$ global symmetry, we can show that the VEV $\langle H(y) \rangle$ is given, without loss of generality, as

(I) $M^2 - \left(\frac{\theta}{L} \right)^2 > 0$

$$\langle H(y) \rangle = \begin{cases} \frac{v}{\sqrt{2}} e^{i \frac{\theta}{L} y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \text{for } -\pi < \theta < \pi, \\ \frac{v}{\sqrt{2}} e^{i \frac{\pi}{L} y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ or } \frac{v}{\sqrt{2}} e^{-i \frac{\pi}{L} y} \begin{pmatrix} 0 \\ 1 \end{pmatrix} & \text{for } \theta = \pi, \end{cases} \quad (2.13)$$

(II) $M^2 - \left(\frac{\theta}{L} \right)^2 \leq 0$

$$\langle H(y) \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad (2.14)$$

where v is given by

$$\left(\frac{v}{\sqrt{2}} \right)^2 := |\langle H(y) \rangle|^2 = \frac{1}{\lambda} \left(M^2 - \left(\frac{\theta}{L} \right)^2 \right). \quad (2.15)$$

From now on, we will assume the case of (I) $M^2 - \left(\frac{\theta}{L} \right)^2 > 0$.

Now we discuss some properties of the derived VEV in Eq. (2.13). Differently from the SM, the VEV possesses y -position-dependence and its broken phase is realized only in the case of $M^2 - \left(\frac{\theta}{L} \right)^2 > 0$. But like the SM, the squared VEV (2.15) is still constant even though $\langle H(y) \rangle$ depends on y . This means that after v/\sqrt{L} is set as 246 GeV, where the mass dimension of v is 3/2, the same situation as the SM occurs in the electroweak symmetry breaking (EWSB) sector. On the other hand, the y -dependence of the Higgs VEV in Eq. (2.13) as an important consequence for the Yukawa sector. Since the VEV of the Higgs doublet appears linearly in each Yukawa term, the overlap integrals which lead to effective 4D Yukawa couplings will produce non-trivial CP phase in the CKM matrix.

In terms of the VEV and physical Higgs modes $h^{(n)}(x)$, H can be expanded as

$$H(x, y) \rightarrow \sum_{n=-\infty}^{\infty} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \left(v e^{i \frac{\theta}{L} y} \delta_{n,0} + h^{(n)}(x) \frac{1}{\sqrt{L}} e^{i \left(\frac{2\pi n + \theta}{L} \right) y} \right) \end{pmatrix}, \quad (2.16)$$

which obeys the boundary condition (2.2). The physical masses $\mu_{h^{(n)}}$ of the zero mode ($n = 0$) and the Kaluza-Klein (KK) modes ($n \neq 0$) are easily calculated from Eq. (2.1) as follows:

$$\mu_{h^{(n)}}^2 = \begin{cases} 2 \left(M^2 - \left(\frac{\theta}{L} \right)^2 \right) = \lambda v^2 & \text{for } n = 0 \\ 2M^2 + \frac{(\theta + 2\pi n)^2}{2L^2} + \frac{(\theta - 2\pi n)^2}{2L^2} - 3 \left(\frac{\theta}{L} \right)^2 = \lambda v^2 + \frac{(2\pi n)^2}{L^2} & \text{for } n \geq 1 \end{cases}, \quad (2.17)$$

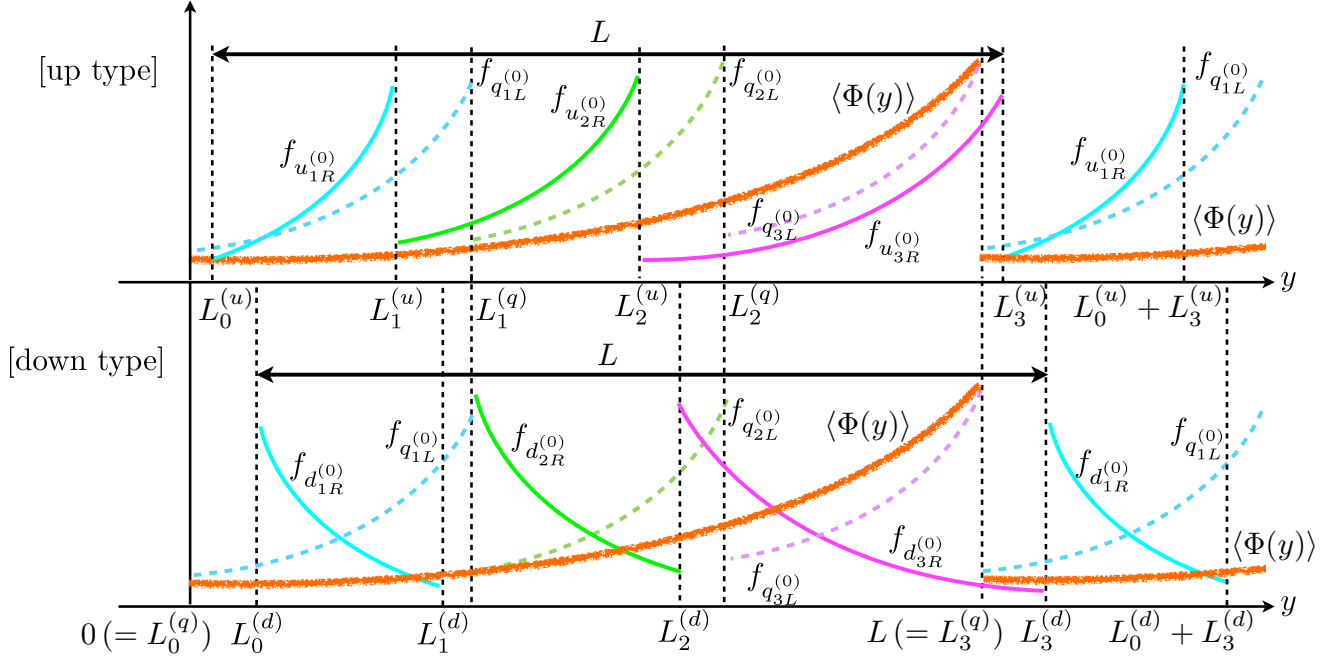


Figure 1: The wavefunction profiles of the quarks and the VEV of $\Phi(y)$ are schematically depicted. Here we take $L_0^{(q)} = L_0^{(\Phi)} = 0$. Note that all the profiles have the periodicity along y with the same period L . Differently from the model on an interval in Ref. [46], we can find the $(1, 3)$ elements of the mass matrices due to the periodicity along y -direction.

with the hermiticity condition for a real field on S^1 : $h^{(n)\dagger} = h^{(-n)}$.

We mention that the relation between $\mu_{h^{(n)}}$ and λ for $n = 0$ in Eq. (2.17) is totally the same as that of the SM. We also comment on the Higgs-quarks couplings in our model. As shown in Eq. (2.16), the profiles of the VEV and the Higgs physical zero mode are the same as $e^{i\frac{\theta}{L}y}$ up to the coefficients. This means that the strengths of the couplings are equivalent to those of the SM even though the mode function gets to be y -position dependent. As a result, the decay branching ratios of the Higgs boson are the same as those of the SM.³

3 The model with point interactions on S^1

Field localization in extra dimensions is known as an effective way of explaining the quark mass hierarchy and pattern of flavor mixing. For this purpose, we follow the strategy in [46], where point interactions are introduced in the bulk space to split and localize fermion profiles and also to produce a y -position-dependent VEV with an (almost) exponential shape, which generates the large fermion mass hierarchy.

There are, however, two different points between the models in this letter and in [46]:

- In the previous model [46], the Higgs VEV cannot possess a non-trivial complex phase, and a CP phase in the CKM matrix has not been realized. On the other hand, the VEV in our

³Being different from the Universal Extra Dimension case [41–45], the “low” KK mass less than a TeV scale is not allowed after considering the level-mixing in the top sector [46]. Then, the significant deviations do not occur in the loop-induced single Higgs production via gluon fusion and Higgs decay processes to a pair of photons and gluons in our model.

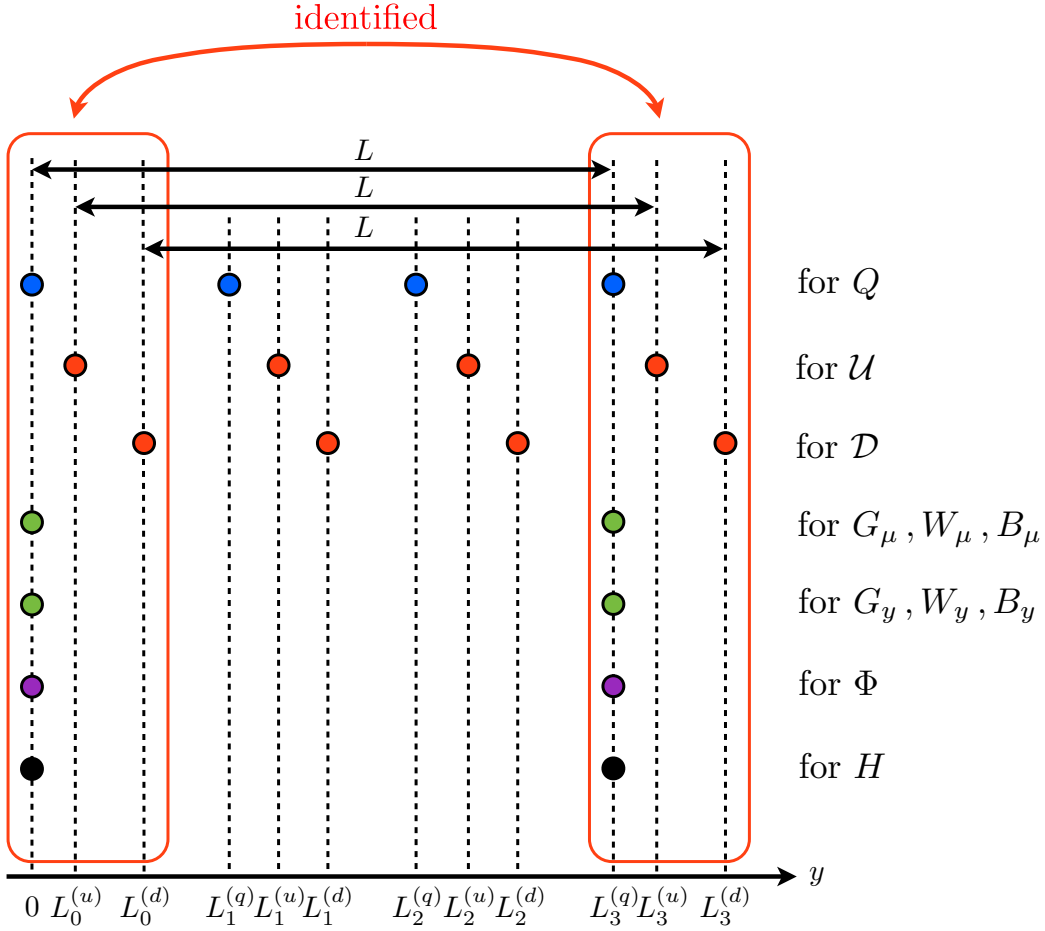


Figure 2: This is an overview of our system with point interactions. The red (blue) circular spots show the Dirichlet BC for left- (right-)handed part at the corresponding boundary points, respectively. The green, purple, and black circular spots represent the ordinary periodic (in Eq. (3.19)), the Robin BCs in Eq. (3.16), and the twisted BCs in Eq. (2.2), respectively. It is noted that we adopt the assumption in Eq. (4.3).

present model has a y -position-dependent complex phase, which will produce a CP phase of the CKM matrix.

- In the previous model [46], the extra dimension has been taken to be an interval, where the twisted BC in Eq. (2.2) cannot be realized. In the present model, we set the extra dimension to be a circle S^1 , whose geometry is compatible with the twisted BC (2.2).

In the following part, we briefly explain how to construct our model. The 5D action for fermions

is given by⁴

$$S = \int d^4x \int_0^L dy \left\{ \left[\bar{Q} (i\partial_M \Gamma^M + M_Q) Q + \bar{\mathcal{U}} (i\partial_M \Gamma^M + M_{\mathcal{U}}) \mathcal{U} + \bar{\mathcal{D}} (i\partial_M \Gamma^M + M_{\mathcal{D}}) \mathcal{D} \right] \right\}, \quad (3.1)$$

where we introduce an $SU(2)_W$ doublet (Q), an up-quark singlet (\mathcal{U}), and a down-type singlet (\mathcal{D}) with the corresponding bulk masses ($M_Q, M_{\mathcal{U}}, M_{\mathcal{D}}$). We note that our model contains only one generation for 5D quarks but each 5D quark produces three generations of the 4D quarks, as we will see below.

We adopt the following BCs for $Q, \mathcal{U}, \mathcal{D}$ with an infinitesimal positive constant ε [46]:

$$Q_R = 0 \quad \text{at} \quad y = L_0^{(q)} + \varepsilon, L_1^{(q)} \pm \varepsilon, L_2^{(q)} \pm \varepsilon, L_3^{(q)} - \varepsilon, \quad (3.2)$$

$$\mathcal{U}_L = 0 \quad \text{at} \quad y = L_0^{(u)} + \varepsilon, L_1^{(u)} \pm \varepsilon, L_2^{(u)} \pm \varepsilon, L_3^{(u)} - \varepsilon, \quad (3.3)$$

$$\mathcal{D}_L = 0 \quad \text{at} \quad y = L_0^{(d)} + \varepsilon, L_1^{(d)} \pm \varepsilon, L_2^{(d)} \pm \varepsilon, L_3^{(d)} - \varepsilon, \quad (3.4)$$

where Ψ_R and Ψ_L denote the eigenstates of γ^5 , i.e. $\Psi_R \equiv \frac{1+\gamma^5}{2}\Psi$ and $\Psi_L \equiv \frac{1-\gamma^5}{2}\Psi$. Here $L_j^{(i)}$ for $i = q, u, d$ and $j = 0, 1, 2, 3$ means the positions of point interactions for the 5D fermions. See Figs. 1 and 2 for details. A crucial consequence of the above BCs is that there appear three-fold degenerated left- (right-)handed zero modes in the mode expansions of Q (\mathcal{U}, \mathcal{D}) and that they form the three generations of the quarks. The details have been given in Ref. [46]. We will not repeat the discussions here.

The fields $Q, \mathcal{U}, \mathcal{D}$ with the BCs in Eqs (3.2)–(3.4) are KK-decomposed as follows:

$$Q(x, y) = \begin{pmatrix} U(x, y) \\ D(x, y) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^3 u_{iL}^{(0)}(x) f_{q_{iL}}^{(0)}(y) \\ \sum_{i=1}^3 d_{iL}^{(0)}(x) f_{q_{iL}}^{(0)}(y) \end{pmatrix} + (\text{KK modes}), \quad (3.5)$$

$$\mathcal{U}(x, y) = \sum_{i=1}^3 u_{iR}^{(0)}(x) f_{u_{iR}}^{(0)}(y) + (\text{KK modes}), \quad (3.6)$$

$$\mathcal{D}(x, y) = \sum_{i=1}^3 d_{iR}^{(0)}(x) f_{d_{iR}}^{(0)}(y) + (\text{KK modes}). \quad (3.7)$$

Here the zero mode functions are obtained in the following forms:

$$f_{q_{iL}}^{(0)}(y) = \mathcal{N}_i^{(q)} e^{M_Q(y-L_{i-1}^{(q)})} \left[\theta(y - L_{i-1}^{(q)}) \theta(L_i^{(q)} - y) \right] \quad \text{in } [L_0^{(q)}, L_3^{(q)}], \quad (3.8)$$

$$f_{u_{iR}}^{(0)}(y) = \mathcal{N}_i^{(u)} e^{-M_{\mathcal{U}}(y-L_{i-1}^{(u)})} \left[\theta(y - L_{i-1}^{(u)}) \theta(L_i^{(u)} - y) \right] \quad \text{in } [L_0^{(u)}, L_3^{(u)}], \quad (3.9)$$

$$f_{d_{iR}}^{(0)}(y) = \mathcal{N}_i^{(d)} e^{-M_{\mathcal{D}}(y-L_{i-1}^{(d)})} \left[\theta(y - L_{i-1}^{(d)}) \theta(L_i^{(d)} - y) \right] \quad \text{in } [L_0^{(d)}, L_3^{(d)}], \quad (3.10)$$

where

$$\Delta L_i^{(l)} = L_i^{(l)} - L_{i-1}^{(l)} \quad (\text{for } i = 1, 2, 3; l = q, u, d), \quad (3.11)$$

⁴ We adopt the representations of the gamma matrices are $\Gamma_\mu = \gamma_\mu$, $\Gamma_y = \Gamma^y = -i\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$ and the Clifford algebra is defined as $\{\Gamma_M, \Gamma_N\} = -2\eta_{MN}$.

$$\mathcal{N}_i^{(q)} = \sqrt{\frac{2M_Q}{e^{2M_Q\Delta L_i^{(q)}} - 1}}, \quad \mathcal{N}_i^{(u)} = \sqrt{\frac{2M_U}{1 - e^{-2M_U\Delta L_i^{(u)}}}}, \quad \mathcal{N}_i^{(d)} = \sqrt{\frac{2M_D}{1 - e^{-2M_D\Delta L_i^{(d)}}}}. \quad (3.12)$$

$\mathcal{N}_i^{(q)}, \mathcal{N}_i^{(u)}, \mathcal{N}_i^{(d)}$ are the wavefunction normalization factors for $f_{q_{iL}}^{(0)}, f_{u_{iL}}^{(0)}, f_{d_{iL}}^{(0)}$, respectively.

Since the length of the total system is universal, $L_3^{(l)} - L_0^{(l)}$ ($l = q, u, d$) should be equal to the circumference of S^1 , i.e.

$$L := L_3^{(q)} - L_0^{(q)} = L_3^{(u)} - L_0^{(u)} = L_3^{(d)} - L_0^{(d)}. \quad (3.13)$$

Note that all the mode functions in Eqs. (3.8)–(3.10) (and a form of a singlet VEV in Eq. (3.17)) are periodic with the common period L , whereas we do not indicate that thing explicitly in Eqs. (3.8)–(3.10).

In this model, the large mass hierarchy is naturally explained with the following Yukawa sector

$$S_Y = \int d^4x \int_0^L dy \left\{ \Phi \left[-\mathcal{Y}^{(u)} \bar{Q}(i\sigma_2 H^*) \mathcal{U} - \mathcal{Y}^{(d)} \bar{Q} H \mathcal{D} \right] + \text{h.c.} \right\}, \quad (3.14)$$

where $\mathcal{Y}^{(u)}/\mathcal{Y}^{(d)}$ is the Yukawa coupling for up/down type quark; H and Φ are an $SU(2)_W$ scalar doublet and a singlet. It should be noted that although the Yukawa couplings $\mathcal{Y}^{(u)}$ and $\mathcal{Y}^{(d)}$ can be complex, they cannot be an origin of the CP phase of the CKM matrix because our model contains only a single quark generation, so that the number of the 5D Yukawa couplings is not enough to produce a CP phase in the CKM matrix. An outline of our system is depicted in Fig. 1. Note that the five terms of $\bar{Q}(i\sigma_2 H^*) \mathcal{U}, \bar{Q} H \mathcal{D}, \Phi \bar{Q} Q, \Phi \bar{\mathcal{U}} \mathcal{U}, \Phi \bar{\mathcal{D}} \mathcal{D}$ with the Pauli matrix σ_2 are excluded by introducing a discrete symmetry $H \rightarrow -H, \Phi \rightarrow -\Phi$. Φ is a gauge singlet and there is no problem with gauge universality violation.⁵

The 5D action and the BCs for Φ are assumed to be of the form [46, 47]

$$S_\Phi = \int d^4x \int_0^L dy \left\{ \Phi^\dagger (\partial_M \partial^M - M_\Phi^2) \Phi - \frac{\lambda_\Phi}{2} (\Phi^\dagger \Phi)^2 \right\}, \quad (3.15)$$

$$\begin{aligned} \Phi + L_+ \partial_y \Phi &= 0 & \text{at } y &= L_0^{(\Phi)} + \varepsilon, \\ \Phi - L_- \partial_y \Phi &= 0 & \text{at } y &= L_3^{(\Phi)} - \varepsilon, \end{aligned} \quad (3.16)$$

where M_Φ (λ_Φ) is the bulk mass (quartic coupling) of the scalar singlet Φ and L_\pm can take values in the range of $-\infty \leq L_\pm \leq \infty$ and $L_0^{(\Phi)}$ and $L_3^{(\Phi)}$ indicate the locations of the two “end points” of the singlet.

The VEV of Φ with the BCs, named Robin BCs, in Eq. (3.16) is expressed in terms of Jacobi’s elliptic functions in general and its phase structure has been discussed in Ref [47]. We adopt a specific form in the region $[L_0^{(\Phi)} + \varepsilon, L_3^{(\Phi)} - \varepsilon]$ [46]:

$$\langle \Phi(y) \rangle = \left[\frac{M_\Phi}{\sqrt{\lambda_\Phi}} \left\{ \sqrt{1+X} - 1 \right\}^{1/2} \right] \times \frac{1}{\text{cn} \left(M_\Phi \{1+X\}^{1/4} (y - y_0), \sqrt{\frac{1}{2} \left(1 + \frac{1}{\sqrt{1+X}} \right)} \right)}, \quad (3.17)$$

⁵ If there exists the doublet-singlet mixing term $-CH^\dagger H \Phi^\dagger \Phi$ with a coefficient C , which cannot be prohibited by the discrete symmetry $H \rightarrow -H, \Phi \rightarrow -\Phi$ in our theory, gauge universality violation should be revisited. A bound from the universality in Z boson gauge couplings was already calculated as $CL \lesssim 0.003$ (when a KK scale is around a few TeV) in a model on an interval [46]. In this letter, we simply ignore this term.

with

$$X := \frac{4\lambda_\Phi |Q|}{M_\Phi^4}. \quad (3.18)$$

Here y_0 and Q are parameters which appear after integration on y and we focus on the choice of $Q < 0$. We note that the values of y_0 and Q are automatically determined after choosing those of L_\pm . As shown in Ref. [46], we get the form of $\langle \Phi(y) \rangle$ to be an (almost) exponential function of y by choosing suitable parameter configurations. Although there is a discontinuity in the wavefunction profile of $\langle \Phi \rangle$ between $y = L_0^{(\Phi)} + \varepsilon$ and $y = L_3^{(\Phi)} - \varepsilon$ in Eqs. (3.16), this type of BCs is derived from the variational principle on S^1 and leads to no inconsistency [47].

The BCs for the 5D $SU(3)_C, SU(2)_W, U(1)_Y$ gauge bosons G_M, W_M, B_M are selected as

$$G_M|_{y=0} = G_M|_{y=L}, \quad \partial_y G_M|_{y=0} = \partial_y G_M|_{y=L}, \quad (3.19)$$

where we only show the G_M 's case. In this configuration, we obtain the SM gauge bosons in zero modes. Based on the discussion in Section 2, we conclude that the W and Z bosons become massive and their masses are suitably created through “our” Higgs mechanism as $m_W \simeq 81 \text{ GeV}$, $m_Z \simeq 90 \text{ GeV}$. The overview of the BCs is summarized in Fig. 2. We mention that, on S^1 geometry, $G_y^{(0)}$, $W_y^{(0)}$, and $B_y^{(0)}$ would exist as massless 4D scalars at the tree level, but they will become massive via quantum corrections and are expected to be uplifted to near KK states. We will discuss those modes in another paper. We should note that in our model on S^1 with point interactions, the 5D gauge symmetries are intact under the BCs summarized in Fig. 2.⁶ Hence the unitarity in the scattering processes of massive particles are ensured in our model.⁷

4 Results

In this section, we would like to find a set of parameter configurations in which the quark mass hierarchy and the structure of the CKM matrix with the CP phase are derived naturally. In the following analysis, we rescale all the dimensional valuables by the S^1 circumference L to make them dimensionless and the rescaled valuables are indicated with the tilde $\tilde{}$.

We set the parameters concerning the scalar singlet Φ as

$$\tilde{M}_\Phi = 8.67, \quad \tilde{y}_0 = -0.1, \quad \tilde{\lambda}_\Phi = 0.001, \quad |\tilde{Q}| = 0.001, \quad (4.1)$$

where the VEV profile becomes an (almost) exponential function of y , which is suitable for generating the large mass hierarchy.⁸ In this case, the values of L_\pm in Eq. (3.16) correspond to

$$\frac{1}{\tilde{L}_+} = -6.07, \quad \frac{1}{\tilde{L}_-} = 8.69, \quad (4.2)$$

where the broken phase is realized [46].

⁶ In Refs. [48–50], the 5D gauge invariance has been discussed from a quantum mechanical supersymmetry point of view.

⁷ Some related works are found in Refs. [51–58].

⁸ The smallness of Q is not an unnatural thing, because they are resultant values derived from the two input parameters L_\pm , whose dimensionless values are within $\mathcal{O}(10)$ as in Eq. (4.2). We note that λ_Φ always appears in the form of the singlet VEV in Eq. (3.17) as the combination $|Q|\lambda_\Phi$. λ_Φ in itself only affects the overall normalization. Therefore some room might remain for more “natural” choice of λ_Φ .

| $m_{ij}^{(u)}$ | a | b | $m_{ij}^{(d)}$ | a | b |
|----------------|-------------|-----------------|----------------|-------------|-----------------|
| $m_{11}^{(u)}$ | $L_0^{(u)}$ | $L_1^{(u)}$ | $m_{11}^{(d)}$ | $L_0^{(d)}$ | $L_1^{(d)}$ |
| $m_{22}^{(u)}$ | $L_1^{(q)}$ | $L_2^{(u)}$ | $m_{22}^{(d)}$ | $L_1^{(q)}$ | $L_2^{(d)}$ |
| $m_{33}^{(u)}$ | $L_2^{(q)}$ | L | $m_{33}^{(d)}$ | $L_2^{(q)}$ | L |
| $m_{12}^{(u)}$ | $L_1^{(u)}$ | $L_1^{(q)}$ | $m_{12}^{(d)}$ | $L_1^{(d)}$ | $L_1^{(q)}$ |
| $m_{23}^{(u)}$ | $L_2^{(u)}$ | $L_2^{(q)}$ | $m_{23}^{(d)}$ | $L_2^{(d)}$ | $L_2^{(q)}$ |
| $m_{31}^{(u)}$ | L | $L + L_0^{(u)}$ | $m_{31}^{(d)}$ | L | $L + L_0^{(d)}$ |

Table 1: The summary table for the overlap integrals in Eq. (4.6).

As in the previous analysis [46], the signs of the fermion bulk masses are assigned as $M_Q > 0$, $M_U < 0$, $M_D > 0$ to make much larger overlapping in up quark sector than in down ones for top mass. Here we assume the positions of the two “end points” of both the quark doublet and the scalar singlet are the same

$$L_0^{(q)} = L_0^{(\Phi)} = 0, \quad L_3^{(q)} = L_3^{(\Phi)} = L, \quad (4.3)$$

where we set $L_0^{(q)}$ and $L_0^{(\Phi)}$ as zero. In addition, we also assume that the orders of the positions of point interactions are settled as

$$\begin{aligned} 0 < L_0^{(u)} < L_1^{(u)} < L_1^{(q)} < L_2^{(u)} < L_2^{(q)} < L < L_3^{(u)}, \\ 0 < L_0^{(d)} < L_1^{(d)} < L_1^{(q)} < L_2^{(d)} < L_2^{(q)} < L < L_3^{(d)}. \end{aligned} \quad (4.4)$$

Here our up quark mass matrix $\mathcal{M}^{(u)}$ and that of down ones $\mathcal{M}^{(d)}$ take the forms

$$\mathcal{M}^{(u)} = \begin{bmatrix} m_{11}^{(u)} & m_{12}^{(u)} & m_{13}^{(u)} \\ 0 & m_{22}^{(u)} & m_{21}^{(u)} \\ 0 & 0 & m_{33}^{(u)} \end{bmatrix}, \quad \mathcal{M}^{(d)} = \begin{bmatrix} m_{11}^{(d)} & m_{12}^{(d)} & m_{13}^{(d)} \\ 0 & m_{22}^{(d)} & m_{21}^{(d)} \\ 0 & 0 & m_{33}^{(d)} \end{bmatrix}, \quad (4.5)$$

where the row (column) index of the mass matrices shows the generations of the left- (right-)handed fermions, respectively. Differently from the model on an interval in Ref. [46], the $(1, 3)$ elements of the mass matrices are allowed geometrically due to the periodicity along y -direction. The general form of the nonzero matrix elements of $\mathcal{M}^{(u)}$ and $\mathcal{M}^{(d)}$ can be expressed as follows:

$$m_{ij}^{(\kappa)} = \mathcal{Y}^{(\kappa)} \int_a^b dy f_{q_{iL}}^{(\kappa)}(y) f_{\kappa_{jR}}^{(\kappa)}(y) \langle \Phi(y) \rangle \langle H(y) \rangle, \quad (4.6)$$

where κ indicates up/down type of quark and the concrete information is stored in Table 1.

The parameters which we use for calculation are

$$\begin{aligned} \tilde{L}_0^{(q)} &= 0, & \tilde{L}_1^{(q)} &= 0.298, & \tilde{L}_2^{(q)} &= 0.659, & \tilde{L}_3^{(q)} &= 1, \\ \tilde{L}_0^{(u)} &= 0.0245, & \tilde{L}_1^{(u)} &= 0.0260, & \tilde{L}_2^{(u)} &= 0.520, & \tilde{L}_3^{(u)} &= 1.03, \\ \tilde{L}_0^{(d)} &= 0.0703, & \tilde{L}_1^{(d)} &= 0.178, & \tilde{L}_2^{(d)} &= 0.646, & \tilde{L}_3^{(d)} &= 1.07, \\ \tilde{M}_Q &= 0.654, & \tilde{M}_U &= -0.690, & \tilde{M}_D &= 0.595, & \theta &= 3.0, \end{aligned} \quad (4.7)$$

where the twist angle θ is a dimensionless value and should be within the range $-\pi < \theta \leq \pi$. We should remind that in our system, the EWSB is only realized on the condition of $M^2 - \left(\frac{\theta}{L}\right)^2 > 0$

as in Eqs. (2.13). Recently, the ATLAS and CMS experiments have announced that the physical Higgs mass is around 126 GeV with 5σ confidence level [59, 60]. $\tilde{\lambda}$ is 0.262 irrespective of the value of L , while \tilde{M} is slightly dependent on the value of L as 3.01303 (3.00052) in the case of $M_{\text{KK}} = 2 \text{ TeV}$ ($M_{\text{KK}} = 10 \text{ TeV}$), where M_{KK} is a typical scale of the KK mode and defined as $2\pi/L$. Here some tuning is required to obtain the suitable values realizing the EWSB.

After the diagonalization of the two mass matrices, the quark masses are evaluated as

$$\begin{aligned} m_{\text{up}} &= 2.06 \text{ MeV}, & m_{\text{charm}} &= 1.25 \text{ GeV}, & m_{\text{top}} &= 174 \text{ GeV}, \\ m_{\text{down}} &= 4.91 \text{ MeV}, & m_{\text{strange}} &= 102 \text{ MeV}, & m_{\text{bottom}} &= 4.18 \text{ GeV}, \\ \frac{m_{\text{up}}}{m_{\text{up}}|_{\text{exp.}}} &= 0.897, & \frac{m_{\text{charm}}}{m_{\text{charm}}|_{\text{exp.}}} &= 0.978, & \frac{m_{\text{top}}}{m_{\text{top}}|_{\text{exp.}}} &= 1.00, \\ \frac{m_{\text{down}}}{m_{\text{down}}|_{\text{exp.}}} &= 1.02, & \frac{m_{\text{strange}}}{m_{\text{strange}}|_{\text{exp.}}} &= 1.07, & \frac{m_{\text{bottom}}}{m_{\text{bottom}}|_{\text{exp.}}} &= 1.00, \end{aligned} \quad (4.8)$$

and the absolute values of the CKM matrix elements are given as⁹

$$|V_{\text{CKM}}| = \begin{bmatrix} 0.971 & 0.238 & 0.00318 \\ 0.238 & 0.970 & 0.0372 \\ 0.00829 & 0.0364 & 0.999 \end{bmatrix}, \quad \left| \frac{V_{\text{CKM}}}{V_{\text{CKM}}|_{\text{exp.}}} \right| = \begin{bmatrix} 0.997 & 1.06 & 0.906 \\ 1.06 & 0.997 & 0.902 \\ 0.957 & 0.900 & 1.00 \end{bmatrix}. \quad (4.9)$$

The Jarlskog parameter J containing information about the CP phase is defined by

$$\text{Im} [(V_{\text{CKM}})_{ij}(V_{\text{CKM}})_{kl}(V_{\text{CKM}}^*)_{il}(V_{\text{CKM}}^*)_{kj}] = J \sum_{m,n=1}^3 \epsilon_{ikm} \epsilon_{jln} \quad (4.10)$$

with the completely antisymmetric tensor ϵ , and is invariant under the $U(1)$ unphysical re-phasing operations of six types of quarks [61, 62]. This value is easily estimated as

$$J = 2.56 \times 10^{-5}, \quad \frac{J}{J|_{\text{exp.}}} = 0.865, \quad (4.11)$$

where we also provide the differences from the latest experimental values in Ref. [63]. All the deviations from the latest experimental values are within about 15% and we can conclude that the situation of the SM is suitably generated.

5 Summary and discussion

In this letter, we proposed a new mechanism for generating CP phase via the Higgs vacuum expectation value (VEV) originating from geometry of an extra dimension. A twisted boundary condition (BC) for the Higgs doublet has been found to lead to an extra dimensional coordinate-dependent VEV with a non-trivial CP phase degree of freedom. This mechanism is useful for realizing CP violation in an extra-dimensional model.

As an application of this idea, we have constructed a phenomenological model with an extra dimension which can simultaneously and naturally explain the origin of the fermion generations, the quark mass hierarchy, and the CKM structure with the CP phase based on [46]. The point interactions realize the three fermion generations and the situation where all the quark profiles are

⁹ The values of $\mathcal{Y}^{(u)}$ and $\mathcal{Y}^{(d)}$ are also chosen as $\mathcal{Y}^{(u)} = -0.0474 + 0.0140 i$ and $\mathcal{Y}^{(d)} = -0.00340 - 0.00152 i$.

split and localized. With the help of the almost exponential function of the scalar VEV, which appears in the Yukawa sector, we can generate the phenomenologically-desirable circumstances where all the flavor structures are realized with good precision and almost all dimensionless scaled parameters take values of natural $\mathcal{O}(10)$ magnitudes.

One of the most important remaining tasks is to construct a model which can explain both of the quark and lepton flavor structures simultaneously. Then, it is necessary to explain why the neutrino masses are so light and the flavor mixings in the lepton sector are large. The result will be reported elsewhere.

Another important topics is the stability of the system. Our system is possibly threatened with instability. Some mechanisms will be required to stabilize the moduli representing the positions of point interactions (branes).¹⁰ In a multiply-connected space of S^1 , there is another origin of gauge symmetry breaking i.e. the Hosotani mechanism [67, 68]. Since further gauge symmetry breaking causes a problem in our model, we need to insure that the Hosotani mechanism does not occur. To this end, we might introduce additional 5D matter to prevent zero modes of y -components of gauge fields from acquiring non-vanishing VEVs. We will leave those issues in future work.

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¹⁰ Moduli stabilization via Casimir energy in the system where a scalar takes the Robin BCs (but no point interaction in the bulk) has been studied in Refs.[64–66].

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